

ON THE MAXIMALITY OF CLASSICAL LOGIC

MARIO PIAZZA AND GABRIELE PULCINI

As is well known, classical propositional logic LK_0 is Post-complete, or maximal: whenever a nontautological formula α is added to it as a new axiom schema, the extended system LK_0^α becomes inconsistent. In other words, the only nontrivial extensions of LK_0 are by *proper* axioms, i.e. formulas that are not closed under uniform substitution. In [2] such extensions of LK_0 are called *supraclassical*. Although cut elimination does not hold in general for supraclassical logics [1] or, it does, but without necessarily entailing the subformula property [3], we show how to fill the gap between classical and supraclassical systems for the propositional fragment. In particular, we show how to *decompose* a proper axiom α into a finite set of atomic, classically underivable, sequents \mathcal{S}_α such that:

- (1) $LK_0^{\mathcal{S}_\alpha}$ enjoys both cut-elimination and subformula property,
- (2) LK_0^α is consistent if, and only if, the empty sequent ‘ \vdash ’ is not in \mathcal{S}_α ,
- (3) \mathcal{S}_α is the *minimal* axiomatic decomposition allowing cut elimination.

We conclude by showing a way to make extensions infinite while preserving nontriviality.

REFERENCES

- [1] Jean-Yves Girard. *Proof theory and logical complexity. Vol. 1*. Bibliopolis, Napoli, 1987.
- [2] David Makinson. Bridges between classical and nonmonotonic logic. *Logic Journal of the IGPL*, 11(1):69–96, 2003.
- [3] Sara Negri and Jan von Plato. Cut elimination in the presence of axioms. *Bulletin of Symbolic Logic*, 4(4):418–435, 1998.

DEPARTMENT OF PHILOSOPHY — UNIVERSITY OF CHIETI-PESCARA
E-mail address, M. Piazza: `mpiazza@unich.it`

CLE — STATE UNIVERSITY OF CAMPINAS
E-mail address, G. Pulcini: `gab.pulcini@cle.unicamp.br`