The Russell-Prawitz translation and schematic rules

Luiz Carlos Pereira – PUC-Rio/UERJ
Edward Hermann Haeusler – PUC-Rio

Translations have been put to many uses in logic. In the first half of last century, translations played an important role in foundational matters: the reduction of foundational questions in classical environments to foundational questions in constructive environments. In the sixties translations acquired theoretical autonomy: several general approaches to translations were proposed (proof-theoretical, algebraic). Another distinctive use of translation is related to schematic rules of inference. What´s an introduction rule for an operator ϕ? What´s an elimination rule for ϕ? In 1978 Dag Prawitz proposed an answer to these questions by means of schematic introduction and elimination rules. Prawitz also proposed a constructive version of the well-known classical truth-functional completeness: if the introduction and elimination rules for an operator ϕ are instances of the schematic introduction and elimination rules, then ϕ is intutionistically definable. Peter Schroeder-Heister showed how to obtain this completeness result for generalized schemata. These schematic introduction and elimination rules can also be used to show that logics whose operators satisfy the schematic rules and whose derivations satisfy the sub-formula principle can be translate into minimal implicational logic. Fernando Ferreira and Gilda Ferreira proposed still another use for translations: to use the Russell-Prawitz translation in order to study the proof-theory of intuitionistic propositional and first-order logic. This study is done in the system *Fat* for atomic polymorphism. This system can be characterized as a second order propositional logic in the language $\{\forall_1, \forall_2, \rightarrow\}$ such that $\forall_2$-elimination is restricted to atomic instantiations. The aim of the present paper is twofold: [1] to use atomic polymorphism to study the proof theory of schematic systems and [2] to produce high-level translations for a large class of logics.