Seligman-style deduction for hybrid modal logic

Patrick Blackburn, Torben Braüner*, Klaus Frovin Jørgensen
Roskilde University, Denmark
and
Thomas Bolander
Technical University of Denmark

A number of different sorts of proof-systems for modal logics are available: Major examples are labelled systems, display logics, multible sequents, and hybrid-logical systems. It is not the purpose of this talk to discuss the pros and cons of all these systems, but instead to focus on one particular sort of proof-system based on hybrid modal logic, which is an extension of ordinary modal logic allowing explicit reference to individual points in a Kripke model (the points stand for times, locations, states, possible worlds, or something else).

This sort of proof-system was put forward by Jerry Seligman in the late 1990s [4]. One particular feature of this sort of system is the possibility to jump to a hypothetical time (or whatever the points stand for), do some reasoning, and then jump back to the present time again. In natural deduction versions [2] the hypothetical reasoning is kept track of using machinery called explicit substitutions, similar to "proof-boxes" in the style of linear logic. Such a hybrid-logical proof-box encapsulates hypothetical reasoning taking place at one particular time. Within the proof-box, information depending on the hypothetical time can be dealt with, but only non-indexical information, that is, statements whose truth-values do not depend on the time, can flow in and out of the box.

In my talk I'll present Seligman-style natural deduction, and I'll in particular discuss the above mentioned machinery enabling hypothetical reasoning. I'll also briefly describe two other lines of work in the area of Seligman-style reasoning:
1. Seligman-style natural deduction has turned out to be useful for formalizing so-called false-belief tests in cognitive psychology [3].
2. Recently developed Seligman-style tableau systems with desirable proof-theoretic properties [1].

References

[1] P. Blackburn, T. Braüner, T. Bolander, and K.F. Jørgensen. A Seligman-style tableau system. In K. McMillan, A. Middeldorp, and A. Voronkov, editors, *Logic for Programming, Artificial Intelligence, and Reasoning*, volume 8312 of *Lecture Notes in Computer Science*, pages 147–163. Springer Publishing Company, 2013.

[2] T. Braüner. *Hybrid Logic and its Proof-Theory*, volume 37 of *Applied Logic Series*. Springer, 2011.

[3] T. Braüner. Hybrid-logical reasoning in false-belief tasks. In B.C. Schipper, editor, *Proceedings of Fourteenth Conference on Theoretical Aspects of Rationality and Knowledge (TARK)*, pages 186–195, 2013. ISBN 978-0-615-74716-3, available at http://tark.org.

[4] J. Seligman. The logic of correct description. In M. de Rijke, editor, *Advances in Intensional Logic*, volume 7 of *Applied Logic Series*, pages 107 – 135. Kluwer, 1997.

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