Proofs and Dialogue : the Ludics view

Alain Lecomte

Laboratoire : “Structures formelles du langage”, Paris 8 Université

February, 2011
Tübingen
with collaboration of Myriam Quatrini

Table of Contents

Where Ludics come from?

Ludics is a theory elaborated by J-Y. Girard in order to rebuild logic starting from the notion of *interaction*.
It starts from the concept of **proof**, as was investigated in the framework of **Linear Logic**:

- **Linear Logic** *may be polarized* ($\rightarrow$ negative and positive rules)
- **Linear Logic** leads to the important notion of **proof-net** ($\rightarrow$ *being a proof is more a question of connections than a question of formulae to be proven*) $\rightarrow$ **loci**

Polarization

Results on polarization come from those on **focalization** (Andréoli, 1992)

- some connectives are *deterministic* and *reversible* (= **negative** ones) : their right-rule, which may be read in both directions, may be applied in a deterministic way:

Example

$$\frac{\vdash A, B, \Gamma}{\vdash A \wp B, \Gamma}\,[\wp] \qquad \frac{\vdash A, \Gamma \quad \vdash B, \Gamma}{\vdash A \& B, \Gamma}\,[\&]$$

Polarization

- the other connectives are *non-deterministic* and *non-reversible* (= **positive** ones) : their right-rule, which cannot be read in both directions, may not be applied in a deterministic way (from bottom to top, there is a choice to be made) :

Example

$$\frac{\vdash A, \Gamma \quad \vdash B, \Gamma'}{\vdash A \otimes B, \Gamma, \Gamma'}[\otimes] \qquad \frac{\vdash A, \Gamma}{\vdash A \oplus B, \Gamma}[\oplus_g] \qquad \frac{\vdash B, \Gamma}{\vdash A \oplus B, \Gamma}[\oplus_d]$$

The Focalization theorem

- every proof may be put in such a form that :
  - as long as there are negative formulae in the (one-sided) sequent to prove, choose one of them at random,
  - as soon as there are no longer negative formulae, choose a positive one and then continue to **focalize** it
- we may consider positive and negative “blocks” $\longrightarrow$ **synthetic connectives**
- convention : the *negative* formulae will be written as positive but on the left hand-side of a sequent $\rightarrow$ **fork**

Hypersequentialized Logic

Formulae:

$$F = O|1|P|(F^{\perp} \otimes \cdots \otimes F^{\perp}) \oplus \cdots \oplus (F^{\perp} \otimes \cdots \otimes F^{\perp})|$$

Rules :

- axioms :

$$\frac{}{P \vdash P, \Delta} \qquad \frac{}{\vdash 1, \Delta} \qquad \frac{}{O \vdash \Delta}$$

- logical rules :

$$\frac{\vdash A_{11}, \ldots, A_{1n_1}, \Gamma \quad \ldots \quad \vdash A_{p1}, \ldots, A_{pn_p}, \Gamma}{(A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}) \vdash \Gamma}$$

$$\frac{A_{i1} \vdash \Gamma_1 \quad \ldots A_{in_i} \vdash \Gamma_p}{\vdash (A_{11}^{\perp} \otimes \cdots \otimes A_{1n_1}^{\perp}) \oplus \cdots \oplus (A_{p1}^{\perp} \otimes \cdots \otimes A_{pn_p}^{\perp}), \Gamma}$$

where $\cup\Gamma_k \subset \Gamma^1$ and, for $k, l \in \{1, \ldots p\}$, $\Gamma_k \cap \Gamma_l = \emptyset$.

- cut rule :

$$\frac{A \vdash B, \Delta \qquad B \vdash \Gamma}{A \vdash \Delta, \Gamma}$$

Remarks

- all propositional variables P are supposed to be **positive**
- formulae connected by the positive $\otimes$ and $\oplus$ are **negative** (positive formulae are maximal positive decompositions)
- $(\ldots \otimes \ldots \otimes \ldots) \oplus (\ldots \otimes \ldots \otimes \ldots) \ldots \oplus (\ldots \otimes \ldots \otimes \ldots)$ is not a restriction because of distributivity $((A \oplus B) \otimes C \equiv (A \otimes C) \oplus (B \otimes C))$

Interpretation of the rules

- **Positive** rule :
  1. choose $i \in \{1, ..., p\}$ (a $\oplus$-member)
  2. then decompose the context Γ into disjoint pieces
- **Negative** rule :
  1. nothing to choose
  2. simply enumerates all the possibilities

First interpretation, as **questions** :

- **Positive** rule : to choose a component where to answer
- **Negative** rule : the range of possible answers

The daimon

Suppose we use a rule:

$$\frac{}{\vdash \Gamma}\ (\mathbf{stop}!)$$

for any sequence Γ, *that we use when and only when we cannot do anything else...*

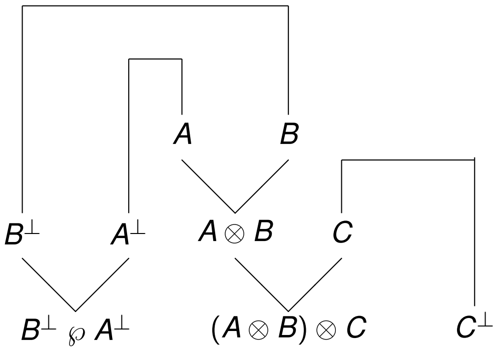
- the system now "accepts" proofs which are not real ones
- if **(stop!)** is used, this is precisely because... the process does not lead to a proof!
- **(stop!)** is a **paralogism**
- the proof ended by **(stop!)** is a **paraproof**
- cf. (in classical logic) it could give a distribution of truth-values which gives a counter-example (therefore also: *counter-proof*)

A reminder of proof-nets

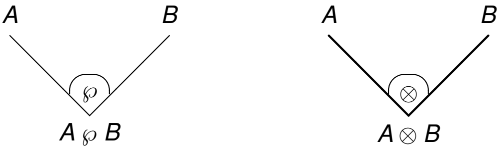
$$\vdash A^{\perp} \wp B^{\perp}, (A \otimes B) \otimes C, C^{\perp}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdash A, A^{\perp} \qquad \vdash B, B^{\perp}}{\vdash A \otimes B, A^{\perp}, B^{\perp}} \qquad \vdash C, C^{\perp}}{\vdash (A \otimes B) \otimes C, A^{\perp}, B^{\perp}, C^{\perp}}}{\vdash A^{\perp}, B^{\perp}, (A \otimes B) \otimes C, C^{\perp}}}{\vdash A^{\perp} \wp B^{\perp}, (A \otimes B) \otimes C, C^{\perp}}$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\vdash A, A^{\perp} \qquad \vdash B, B^{\perp}}{\vdash A \otimes B, A^{\perp}, B^{\perp}}}{\vdash A \otimes B, A^{\perp} \wp B^{\perp}} \qquad \vdash C, C^{\perp}}{\vdash (A \otimes B) \otimes C, A^{\perp} \wp B^{\perp}, C^{\perp}}}{\vdash A^{\perp} \wp B^{\perp}, (A \otimes B) \otimes C, C^{\perp}}$$

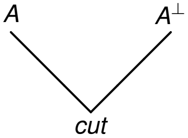

A
B
$B^\perp$
$A^\perp$
$A \otimes B$
C
$B^\perp \wp A^\perp$
$(A \otimes B) \otimes C$
$C^\perp$

1. "par" and "tensor" links:



2. "Axiom" link

3. "Cut" link

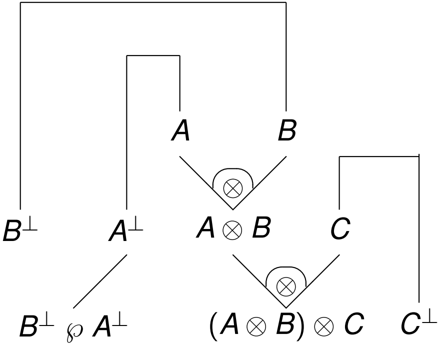


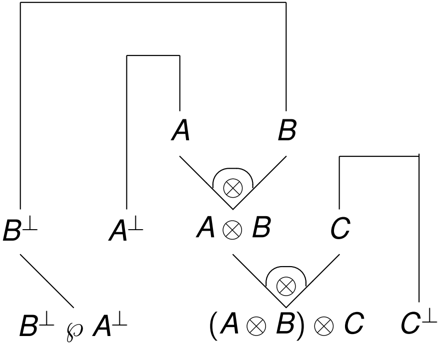
We define a *proof structure* as any such a graph built only by means of these links such that each formula is the conclusion of exactly one link and the premiss of at most one link.

Criterion

Definition (Correction criterion)

correction criterion A proof structure is a proof net if and only if the graph which results from the removal, for each $\wp$ link ("par" link) in the structure, of one of the two edges is connected and has no cycle (that is in fact a tree).


A
B
$B^{\perp}$
$A^{\perp}$
$A \otimes B$
C
$B^{\perp} \wp A^{\perp}$
$(A \otimes B) \otimes C$
$C^{\perp}$


A
B
$B^\perp$
$A^\perp$
$A \otimes B$
C
$B^\perp \wp A^\perp$
$(A \otimes B) \otimes C$
$C^\perp$

Loci

Rules do not apply to **contents** but to **addresses**

Example

$$\dfrac{\vdash e^{\perp}, l \qquad \dfrac{\vdash e^{\perp}, c}{\vdash e^{\perp}, c \oplus d}}{\dfrac{\vdash e^{\perp}, l \& (c \oplus d)}{\vdash e^{\perp} \wp (l \& (c \oplus d))}} \qquad \dfrac{\vdash e^{\perp}, l \qquad \dfrac{\vdash e^{\perp}, d}{\vdash e^{\perp}, c \oplus d}}{\dfrac{\vdash e^{\perp}, l \& (c \oplus d)}{\vdash e^{\perp} \wp (l \& (c \oplus d))}}$$

under a focused format :

$$\dfrac{\vdash e^{\perp}, l \qquad \dfrac{c^{\perp} \vdash e^{\perp}}{\vdash e^{\perp}, c \oplus d}}{e \otimes (l^{\perp} \oplus (c \oplus d)^{\perp}) \vdash} \qquad \dfrac{\vdash e^{\perp}, l \qquad \dfrac{d^{\perp} \vdash e^{\perp}}{\vdash e^{\perp}, c \oplus d}}{e \otimes (l^{\perp} \oplus (c \oplus d)^{\perp}) \vdash}$$

with only loci:

$$\dfrac{\vdash \xi 1, \xi 2 \qquad \dfrac{\xi.3.1 \vdash \xi 1}{\vdash \xi.1, \xi.3}}{\xi \vdash} \qquad\qquad \dfrac{\vdash \xi 1, \xi 2 \qquad \dfrac{\xi.3.2 \vdash \xi 1}{\vdash \xi.1, \xi.3}}{\xi \vdash}$$

Rules

Definition

positive rule

$$\frac{\dots \quad \xi \star i \vdash \Lambda_i \quad \dots}{\vdash \xi, \Lambda}\,(+, \xi, I)$$

- $i \in I$
- all Λ_i's pairwise disjoint and included in Λ

Definition

negative rule

$$\frac{\dots \quad \vdash \xi \star J, \Lambda_J \quad \dots}{\xi \vdash \Lambda}\,(-, \xi, \mathcal{N})$$

daimon

Dai

$$\frac{}{\vdash \Lambda}\dagger$$

- it is a positive rule (something we choose to do)
- it is a *paraproof*

Is there a identity rule?

- No, properly speaking (since there are lo longer **atoms**!)
- two *loci* cannot be identified
- there only remains the opportunity to recognize that two sets of addresses correspond to each other by displacement : $\mathcal{F}ax$

$$Fax_{\xi,\xi'} = \cfrac{\cfrac{\cfrac{\dots \quad Fax_{\xi_{i1},\xi'_{i1}} \quad \dots}{\dots\xi' \star i \vdash \xi \star i \dots}}{\dots \quad \vdash \xi \star J_1, \xi' \quad \dots}\,(+, \xi', J_1)}{\xi \vdash \xi'}\,(-, \xi, \mathcal{P}_f(\mathbb{N}))$$

Infinite proofs

- $\mathcal{F}ax$.... is **infinite**! (cf. the directory $\mathcal{P}_f(\mathbb{N})$)
- it provides a way to explore any “formula” (a tree of addresses) at any depth

Designs

Definition

A **design** is a tree of **forks** $\Gamma \vdash \Delta$ the root of which is called the **base** (or conclusion), which is built only using :

- daimon
- positive rule
- negative rule

a design...

Example

$$
\dfrac{\dfrac{011 \vdash \quad 012 \vdash 02}{\vdash 01, 02}(+, 01, \{1,2\}) \quad \dfrac{031 \vdash \quad 033 \vdash 01}{\vdash 01, 03}(+, 03, \{1,3\})}{\dfrac{0 \vdash}{\vdash <>}(+, <>, \{0\})}(-, 0, \{\{1,2\}, \{1,3\}\})
$$

- a negative step gives a fixed **focus** and a set of **ramifications**,
- on such a basis, a positive step chooses a **focus** and a **ramification**

An illustration

- positive rule : a **question** (*where will you go next week ?*)
- negative rule : a scan of possible answers is provided, (*Roma* and *Naples* or *Rome* and *Florence*)
- in case of the choice 1 : positive rule on the base ”Roma”, new questions (*with whom?* and *by what means?*)
- in case of choice 2 : positive rule on the base ”Florence”, new questions (*with whom?* and *how long will you stay?*)

Normalization

- no explicit cut-rule in Ludics
- but an implicit one : the meeting of same addresses with opposite polarity

Example

$$\dfrac{\dfrac{\dfrac{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \quad \dfrac{\dfrac{\vdots}{\vdash \xi 21} \quad \dfrac{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{\vdash \color{red}{\xi}} \qquad \dfrac{\dfrac{\dfrac{\vdots}{\xi 11 \vdash \xi 2} \quad \dfrac{\vdots}{\xi 12 \vdash}}{\vdash \xi 1, \xi 2}}{\color{red}{\xi} \vdash}$$

$$\overline{\hspace{12em}}$$

$$\dfrac{\dfrac{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \qquad \dfrac{\dfrac{\dfrac{\vdots}{\xi 11 \vdash \xi 2} \quad \dfrac{\vdots}{\xi 12 \vdash}}{\vdash \xi 1, \xi 2} \qquad \dfrac{\dfrac{\vdots}{\vdash \xi 21} \quad \dfrac{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{}$$

which is rewritten in:

$$\dfrac{\begin{matrix}\vdots \\ \xi 12 \vdash\end{matrix} \qquad \begin{matrix}\vdots \\ \vdash \xi 12, \xi 11\end{matrix}}{} \qquad \dfrac{}{\begin{matrix}\vdots \\ \xi 11 \vdash \xi 2\end{matrix}} \qquad \dfrac{}{\dfrac{\begin{matrix}\vdots \\ \vdash \xi 21\end{matrix} \qquad \begin{matrix}\vdots \\ \vdash \xi 22, \xi 23\end{matrix}}{\xi 2 \vdash}}$$

And so on . . .

When the interaction meets the daimon, it converges. The two interacting designs are said **orthogonal**

$$
\cfrac{\cfrac{\cfrac{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \qquad \cfrac{\cfrac{\vdots}{\vdash \xi 21} \quad \cfrac{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{\vdash \xi} \qquad \cfrac{\cfrac{}{\vdash \xi 1, \xi 2}\dagger}{\xi \vdash}
$$

$$
\cfrac{\vdash \xi \qquad \xi \vdash}{}
$$

Otherwise the interaction is said **divergent**.

$$\dfrac{\dfrac{\dfrac{\overset{\vdots}{\vdash \xi 11, \xi 12}}{\xi 1 \vdash} \qquad \dfrac{\overset{\vdots}{\vdash \xi 21} \quad \overset{\vdots}{\vdash \xi 22, \xi 23}}{\xi 2 \vdash}}{\vdash \xi} \qquad \dfrac{\overset{\vdots}{\vdash \xi 1, \xi 2, \xi 3}}{\xi \vdash}}{}$$

Normalization, formally - 1- Closed nets

Namely, a **closed net** consists in a cut between the two following designs:

$$\dfrac{\dfrac{\begin{matrix}\mathcal{D}\\ \vdots\end{matrix}}{\vdash \xi}\,\kappa \qquad \dfrac{\begin{matrix}\mathcal{E}\\ \vdots\end{matrix}}{\xi \vdash}\,(\xi, \mathcal{N})}{}$$

Orthogonality

- if κ is the daimon, then the normalized form is :

$$\frac{}{\vdash} \dagger$$

(this normalised net is called dai)

- if $\kappa = (\xi, I)$, then if $I \notin \mathcal{N}$, normalization fails,
- if $\kappa = (\xi, I)$ and $I \in \mathcal{N}$, then we consider, for all $i \in I$ the design $\mathcal{D}_i$, sub-design of $\mathcal{D}$ of basis $\xi \star i \vdash$, and the sub-design $\mathcal{E}'$ of $\mathcal{E}$, of basis $\vdash \xi \star I$, and we replace $\mathcal{D}$ and $\mathcal{E}$ by, respectively, the sequences of $\mathcal{D}_i$ and $\mathcal{E}'$.

In other words, the initial net is replaced by :

$$\dfrac{\dfrac{\mathcal{D}_{i_1} \atop \vdots}{\xi \star i_1 \vdash} \quad \ldots \quad \dfrac{\mathcal{E}' \atop \vdots}{\vdash \xi \star i_1, \ldots, \xi \star i_n} \quad \dfrac{\mathcal{D}_{i_n} \atop \vdots}{\xi \star i_n \vdash}}{}$$

with a cut between each $\xi \star i_j \vdash$ and the corresponding "formula" $\xi \star i_j$ in the design $\mathcal{E}'$

An example of normalization which does not yield *dai*

$\mathcal{F}ax_{\xi \vdash \rho}$ against a design $\mathcal{D}$ of basis $\vdash \xi$

Let $\mathcal{D}$ the design :

$$\dfrac{\dfrac{\mathcal{D}_1}{\xi \star 1 \vdash} \quad \dfrac{\mathcal{D}_2}{\xi \star 2 \vdash}}{\vdash \xi}$$

Normalization selects first the slice corresponding to $\{1, 2\}$, after elimination of the first cut, it remains:

$$\dfrac{\mathcal{D}_1}{\xi \star 1 \vdash} \qquad \dfrac{\mathcal{D}_2}{\xi \star 2 \vdash} \qquad \dfrac{\dfrac{\mathcal{F}ax}{\rho \star 1 \vdash \xi \star 1} \quad \dfrac{\mathcal{F}ax}{\rho \star 2 \vdash \xi \star 2}}{\vdash \xi \star 1, \xi \star 2, \rho}$$

and finally:

suite

$$\dfrac{\dfrac{\mathcal{D}'_1}{\rho \star 1 \vdash} \quad \dfrac{\mathcal{D}'_2}{\rho \star 2 \vdash}}{\vdash \rho}$$

where, in $\mathcal{D}'_1$ and $\mathcal{D}'_2$, the address ξ is systematically relaced by ρ.

The separation theorem

Theorem

If $\mathcal{D} \neq \mathcal{D}'$ then there exists a counterdesign $\mathcal{E}$ which is orthogonal to one of $\mathcal{D}, \mathcal{D}'$ but not to the other.

Hence the fact that: the objects of ludics are completely defined by their interactions

- a design $\mathcal{D}$ inhabits its **behaviour** (= like its **type**)
- a **behaviour** is a set of designs which is stable by **bi-orthogonality** ($\mathbb{G} = \mathbb{G}^{\perp\perp}$)

The game aspect

A slight change of vocabulary:

step in a proof	action	
positive step	**positive** action	$(+, \xi, I)$
negative step	**negative** action	$(-, \zeta, J)$
branch of a design	*play in a game*	**chronicle**
design	*strategy*	design (**dessein**) as a set of chronicles

Example

$$\dfrac{\dfrac{011 \vdash \quad 012 \vdash 02}{\vdash 01, 02}(+, 01, \{1,2\}) \qquad \dfrac{}{\vdash 01, 03}\dagger}{\dfrac{0 \vdash}{\vdash <>}(+, <>, \{0\})}(-, 0, \{\{1,2\}, \{1,3\}\})$$

Example

$(+, <>, 0), (-, 0, \{1, 2\}), (+, 01, \{1, 2\})$
$(+, <>, 0), (-, 0, \{1, 3\}), (+, \dagger)$

Dialogue in Ludics

The archetypal figure of interaction is provided by two intertwined processes the successive times of which, alternatively positive and negative, are opposed by pairs.

Ludics	**Dialogue**
Positive rule	**performing** an intervention or **commiting oneself** (Brandom)
Negative rule	**recording** or awaiting or **giving authorization**
Daïmon	giving up or ending an exchange

$$\dfrac{\begin{matrix}\vdots & \vdots & \vdots \\ P_1 \vdash \Delta_1 & P_2 \vdash \Delta_2 & P_3 \vdash \Delta_3\end{matrix}}{\vdash \mathbf{P}, \Delta}$$

I **commit myself**
to speak of P_1, P_2, P_3

$$\dfrac{\begin{matrix}\vdots & & \vdots & & \vdots \\ \vdash Q_1, Q_2, \Gamma & \ldots & \vdash R_1, \Gamma & \ldots & \vdash P_1, P_2, P_3, \Gamma\end{matrix}}{\mathbf{P} \vdash \Gamma}$$

among **authorizations**
provided by interlocutor

The daimon rule

$$\frac{}{\vdash \Delta}\dagger$$

- In proof reading this represents the fact to abandon your proof search or your counter-model attempt.
- This represents the fact to close a dialogue (by means of some explicite signs : "well", "OK", . . . or implicitely because it is clear that an answer was given, an argument was accepted and so on. . .).

Convergence and divergence

- *Convergence in dialogue holds as long as commitments of one speaker belong to authorizations provided by the other speaker* (*pragmatics*: *"Be relevant!"* replaced by *"Keep convergent!"*)
- **orthogonality** = **private** communication
- **non-orthogonality** : normalization may yield *side effects* : public results of communication

Examples

Example of two elementary dialogues:

Example

The first one is well formed:

- Have you a car?
- Yes,
- Of what mark?

$$\dfrac{\dfrac{}{\vdash}\dagger \qquad \dfrac{\dfrac{Fax_{\xi 010,\sigma}}{\xi 010 \vdash \sigma}}{\vdash \xi 01, \sigma}}{\dfrac{\xi 0 \vdash \sigma}{\vdash \xi, \sigma}}\{\emptyset,\{1\}\} \qquad \text{vs} \qquad \dfrac{\dfrac{\dfrac{\vdots}{\vdash \xi 010}}{\xi 01 \vdash}}{\dfrac{\vdash \xi 0}{\xi \vdash}}$$

Examples

The locus σ is a place for recording the answer:

Example

- Have you a car?
- Yes,
- Of what mark?
- Honda.

$$
\cfrac{\cfrac{}{\vdash}\dagger \qquad \cfrac{\cfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \sigma}}{\vdash \xi01, \sigma}}{\cfrac{\xi0 \vdash \sigma}{\vdash \xi, \sigma}}\{\emptyset,\{1\}\}
\qquad \text{vs} \qquad
\cfrac{\cfrac{\cfrac{\cfrac{\xi010k \vdash}{\vdash \xi010}}{\xi01 \vdash}}{\vdash \xi0}}{\xi \vdash}
$$

The interaction reduces to:

Example

$$\frac{\sigma k \vdash}{\vdash \sigma}$$

The mark of the car is “Honda”.
This “assertion” is recorded by the speaker.

It is the function of $\mathcal{F}ax$ to interact in such a way that the design anchored on ξ_{010} is transferred to the address σ, **thus providing the answer.**

The second dialogue is ill-formed: *- Have you a car?*

- No, I have no car

*- * Of what mark?*

$$\dfrac{\dfrac{\dfrac{\dfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \xi01, \sigma}}{\vdash \xi01, \sigma}\,You_3}{\xi0 \vdash \sigma}\,\{\{1\}\}}{\vdash \xi, \sigma}\,You_1 \qquad \text{vs} \qquad \dfrac{\dfrac{}{\vdash \xi0}\,\emptyset}{\xi \vdash}$$

$$\dfrac{\dfrac{\dfrac{\xi010 \vdash}{\vdash \sigma} \quad \dfrac{\dfrac{Fax_{\xi010,\sigma}}{\xi010 \vdash \sigma}}{\vdash \xi01, \sigma}\,You_3}{\xi0 \vdash \sigma}\,\{\emptyset,\{1\}\}}{\vdash \xi, \sigma}\,You_1 \qquad \text{vs} \qquad \dfrac{\dfrac{}{\vdash \xi0}\,\emptyset}{\xi \vdash}$$

Modelling dialogue

Intervention of S	**Current state**	**Intervention of** A
$\mathfrak{S}_1$		
	$\mathfrak{E}_1 = \mathfrak{S}_1$	
		$\mathfrak{A}_2$
	$\mathfrak{E}_2 = [[\mathfrak{E}_1, \mathfrak{A}_2]]$	
$\mathfrak{S}_3$		
	$\mathfrak{E}_3 = [[\mathfrak{E}_2, \mathfrak{S}_3]]$	
$\vdots$	$\vdots$	$\vdots$

Rebuilding Logic

- **behaviours**
- operations on behaviours

Example

Additives :

- if $\mathbf{G}$ and $\mathbf{H}$ are two disjoint negative behaviours : $\mathbf{G} \,\&\, \mathbf{H} = \mathbf{G} \cap \mathbf{H}$
- if they are positive $\mathbf{G} \oplus \mathbf{H} = \mathbf{G} \sqcup \mathbf{H}$ $(= (\mathbf{G} \cup \mathbf{H})^{\perp\perp})$

Rebuilding Logic-2

Example

Multiplicatives :

- Let us take two positive designs $\mathcal{D}$ and $\mathcal{D}'$ starting from respectively $(+, \xi, I)$ and $(+, \xi, J)$, we may make a new design starting from $(+, \xi, I \cup J)$. The problem is : what to do with $I \cap J$?
  - we may introduce a priority $\rightarrow$ non-commutative $\otimes$
  - or we may stop those branches by $\mathcal{D}ai_{-}$ (a special design ended by $\dagger$) $\rightarrow \otimes$

Further developments

- K. Terui's c-designs : **computational designs**
  - from *absolute* addresses to relative addresses : *variables* of designs
  - ramifications replaced by named actions with an arity
  - finite objects: generators, in case of infinite designs
  - c-designs are **terms** which generalize λ-terms(simultaneous and parallel reductions via several channels)
- inclusion of **exponentials** (authorizes replay)

The introduction of variables allows to deal with designs with variables which correspond to designs with partial information (the whole future may stay unknown)

Conclusion

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics, he lives in a monist universe
  - proof vs counter - proof
- proofs (**dessins**) and strategies (**desseins**) are two faces of the same objects
- **formulae** (= **types**) are **behaviours**
- behaviours **can be decomposed** by means of &, $\oplus$, $\otimes$, thus providing the analogues of formulae of Linear (or Affine?) Logic
- **no atoms** : such decompositions may be **infinite**!
- this opens the field to considering very ancient conceptions of Logic (**Nāgārjuna**) for which *there are no grounded foundations of our assertions*

Conclusion

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics, he lives in a monist universe
  - proof vs counter - proof
- proofs (**dessins**) and strategies (**desseins**) are two faces of the same objects
- **formulae** (= **types**) are **behaviours**
- behaviours **can be decomposed** by means of &, $\oplus$, $\otimes$, thus providing the analogues of formulae of Linear (or Affine?) Logic
- **no atoms** : such decompositions may be **infinite**!
- this opens the field to considering very ancient conceptions of Logic (**Nāgārjuna**) for which *there are no grounded foundations of our assertions*

Conclusion

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics, he lives in a monist universe
  - proof vs counter - proof
- proofs (**dessins**) and strategies (**desseins**) are two faces of the same objects
- **formulae** (= **types**) are **behaviours**
- behaviours **can be decomposed** by means of &, $\oplus$, $\otimes$, thus providing the analogues of formulae of Linear (or Affine?) Logic
- **no atoms** : such decompositions may be **infinite**!
- this opens the field to considering very ancient conceptions of Logic (**Nāgārjuna**) for which *there are no grounded foundations of our assertions*

Conclusion

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics, he lives in a monist universe
  - proof vs counter - proof
- proofs (**dessins**) and strategies (**desseins**) are two faces of the same objects
- **formulae** (= **types**) are **behaviours**
- behaviours **can be decomposed** by means of &, $\oplus$, $\otimes$, thus providing the analogues of formulae of Linear (or Affine?) Logic
- **no atoms** : such decompositions may be **infinite**!
- this opens the field to considering very ancient conceptions of Logic (**Nāgārjuna**) for which *there are no grounded foundations of our assertions*

Conclusion

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics, he lives in a monist universe
  - proof vs counter - proof
- proofs (**dessins**) and strategies (**desseins**) are two faces of the same objects
- **formulae** (= **types**) are **behaviours**
- behaviours **can be decomposed** by means of &, $\oplus$, $\otimes$, thus providing the analogues of formulae of Linear (or Affine?) Logic
- **no atoms** : such decompositions may be **infinite**!
- this opens the field to considering very ancient conceptions of Logic (**Nāgārjuna**) for which *there are no grounded foundations of our assertions*

Conclusion

- usually, the logician lives in a dualist universe:
  - proof vs (counter) - model
- with ludics, he lives in a monist universe
  - proof vs counter - proof
- proofs (**dessins**) and strategies (**desseins**) are two faces of the same objects
- **formulae** (= **types**) are **behaviours**
- behaviours **can be decomposed** by means of &, $\oplus$, $\otimes$, thus providing the analogues of formulae of Linear (or Affine?) Logic
- **no atoms** : such decompositions may be **infinite**!
- this opens the field to considering very ancient conceptions of Logic (**Nāgārjuna**) for which *there are no grounded foundations of our assertions*