Implications as rules in dialogues

Thomas Piecha

(joint work with Peter Schroeder-Heister)

University of Tübingen
Wilhelm-Schickard-Institute

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Dialogues

A dialogue for $a \to (b \wedge a)$

0.	P	$a \to (b \wedge a)$	
1.	O	a	[0, attack]
2.	P	$b \wedge a$	[1, defense]
3.	O	$\wedge_2$	[2, attack]
4.	P	a	[3, defense]

positions (rows 0–4); *moves* (the player, formula and bracketed columns)

Argumentation forms

X and Y, where $X \neq Y$, are variables for P and O.

implication $\to$:	assertion:	$X\,A \to B$	
	attack:	$Y\,A$	
	defense:	$X\,B$	
conjunction $\wedge$:	assertion:	$X\,A_1 \wedge A_2$	
	attack:	$Y \wedge_i$	(Y chooses $i = 1$ or $i = 2$)
	defense:	$X\,A_i$	

Dialogues

Dialogue (1)

A *dialogue* is a sequence of moves

(i) where P and O take turns,

(ii) according to the argumentation forms,

(iii) and P makes the first move.

Dialogue (2)

(D) P may assert an atomic formula only if it has been asserted by O before.

(E) O can only react on the immediately preceding P-move.

(plus some other conditions)

A dialogue beginning with $P\,A$ is called *dialogue for the formula A*.

Argumentation forms P/O-symmetric.

Asymmetry between proponent P and opponent O due to (D) and (E).

Dialogues

P wins a dialogue

P wins a dialogue for a formula A if

(i) the dialogue is finite,

(ii) begins with the move $P\,A$ and

(iii) ends with a move of P such that O cannot make another move.

Example, dialogue won by P

0.	P	$(a \vee b) \to \neg\neg(a \vee b)$	
1.	O	$a \vee b$	$[0, A]$
2.	P	$\vee$	$[1, A]$
3.	O	a	$[2, D]$
4.	P	$\neg\neg(a \vee b)$	$[1, D]$
5.	O	$\neg(a \vee b)$	$[4, A]$
6.	P	$a \vee b$	$[5, A]$
7.	O	$\vee$	$[6, A]$
8.	P	a	$[7, D]$

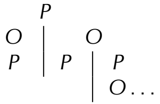
Dialogue not won by P

0.	P	$(a \vee b) \to \neg\neg(a \vee b)$	
1.	O	$a \vee b$	$[0, A]$
2.	P	$\neg\neg(a \vee b)$	$[1, D]$
3.	O	$\neg(a \vee b)$	$[2, A]$
4.	P	$a \vee b$	$[3, A]$

Strategies

Strategy

A *dialogue tree* contains all possible dialogues for A as paths.

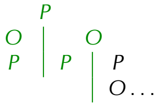


A *strategy* for a formula A is a subtree S of the dialogue tree for A such that

(i) S does not branch at even positions (i.e. at P-moves),

(ii) S has as many nodes at odd positions as there are possible moves for O,

(iii) all branches of S are dialogues for A won by P.



Database perspective: Clausal definitions

Definitional clause (for atomic a)

A *definitional clause* is an expression of the form

$$a \leftarrow B_1 \wedge \ldots \wedge B_n$$

for $n \geqslant 0$, where a is atomic and B_i can be complex.

Clausal definition

A finite set $\mathcal{D}$ of definitional clauses

$$\mathcal{D} \begin{cases} a \leftarrow \Delta_1 \\ \quad \vdots \\ a \leftarrow \Delta_k \end{cases}$$

is a *definition* of a, where $\Delta_i = B^i_1 \wedge \ldots \wedge B^i_{n_i}$ is the body of the i-th clause.

Definitional closure and reflection

For a given definition

$$\mathcal{D} \left\{ \begin{array}{l} a \leftarrow \Delta_1 \\ \vdots \\ a \leftarrow \Delta_k \end{array} \right.$$

we have for sequents:

Principle of definitional closure $(\vdash \mathcal{D})$

$$\frac{\Gamma \vdash \Delta_i}{\Gamma \vdash a}\ (\vdash \mathcal{D})$$

Principle of definitional reflection $(\mathcal{D} \vdash)$

$$(\mathcal{D} \vdash)\ \frac{\Gamma, \Delta_1 \vdash C \quad \ldots \quad \Gamma, \Delta_k \vdash C}{\Gamma, a \vdash C}$$

(for propositional atoms; for first-order a proviso is needed)

Definitional closure and reflection

In sequent calculus:

Proof theory is extended to atomic formulas.

Proofs do not have to begin with atomic formulas.

Implications in definition (database/logic program) read as rules.

Symmetry at level of definitional closure/reflection.

For dialogues:

Add end-rule for complex formulas.

Equivalent to sequent calculus with complex initial sequents.

Extend to (definitional) reasoning for atomic formulas.

P/*O*-symmetry will obtain.

C-dialogues

C-dialogue

A *C-dialogue* is a dialogue with the condition (end-rule)

(C) O can attack a formula A if and only if
(i) A has not yet been asserted by O, or
(ii) A has already been attacked by P.

The notions 'dialogue won by P', 'dialogue tree' and 'strategy' as defined for dialogues are directly carried over to the corresponding notions for C-dialogues.

Difference dialogue / C-dialogue won by P:

(i) Dialogue can only end with assertion of atomic formula,

(ii) whereas C-dialogue ends with assertion of a complex or atomic formula.

C-dialogues

Example, C-strategy for $(a \vee b) \to \neg\neg(a \vee b)$

0.	P	$(a \vee b) \to \neg\neg(a \vee b)$	
1.	O	$a \vee b$	$[0, A]$
2.	P	$\neg\neg(a \vee b)$	$[1, D]$
3.	O	$\neg(a \vee b)$	$[2, A]$
4.	P	$a \vee b$	$[3, A]$

O cannot attack $a \vee b$ since the conditions of (C) are not satisfied:

(i) $a \vee b$ has already been asserted by O and

(ii) $a \vee b$ has not been attacked by P.

The C-dialogue is won by P, and it is a C-strategy for $(a \vee b) \to \neg\neg(a \vee b)$.

C-dialogues

Complex initial sequent

$$(\mathrm{Id})\ \frac{}{A \vdash A}\ \ (A \text{ atomic or complex})$$

Theorem (Isomorphism)

C-strategies and sequent calculus derivations with complex initial sequents are isomorphic.

(Modulo structural inferences, depending on level of precision in trafo.)

(Proof for intuitionistic logic by T. Piecha à la Sørensen/Urzyczyn.)

Important in definitional reasoning where meaning of atomic formulas can be given by complex formulas (corresponds to complex assumptions).

Definitional reasoning

Argumentation form

For each atom a defined by $\mathcal{D}$ $\begin{cases} a \leftarrow \Delta_1 \\ \quad \vdots \\ a \leftarrow \Delta_k \end{cases}$

definitional reasoning:	assertion:	$X\,a$	
	attack:	$Y\,\mathscr{D}$	
	defense:	$X\,\Delta_i$	(X chooses $i = 1, \ldots, k$)

('$\mathscr{D}$' special symbol indicating attack.)

With $O\,\mathscr{D} \simeq$ *definitional closure* $(\vdash \mathcal{D})$; with $P\,\mathscr{D} \simeq$ *definitional reflection* $(\mathcal{D} \vdash)$.

Definitional dialogues

Definitional dialogues are C-dialogues

(i) plus argumenation form of definitional reasoning

(ii) can start with assertion of atomic formula.

Implications as rules: Argumentation forms

assertion:	$O\ A{\to}B$
attack:	*no attack*
defense:	*(no defense)*

assertion:	$O\ A_1{\wedge}A_2$	
attack:	$P\ \wedge_i$	$(i = 1 \text{ or } 2)$
defense:	$O\ A_i$	

assertion:	$P\ A{\to}B$	
question:	$O\ ?$	
choice:	$P\ \lvert A{\to}B \rvert$	$P\ C$ only if $O\ C{\to}(A{\to}B)$ before
attack:	$O\ A$	
defense:	$P\ B$	

assertion:	$P\ A_1{\wedge}A_2$	
question:	$O\ ?$	
choice:	$P\ \lvert A_1{\wedge}A_2 \rvert$	$P\ C$ only if $O\ C{\to}(A_1{\wedge}A_2)$ before
attack:	$O\ \wedge_i$ $(i = 1 \text{ or } 2)$	
defense:	$P\ A_i$	

P/*O*-symmetry of argumentation forms is given up.

Implications as rules: Argumentation forms

assertion:	$O\ A \to B$	
attack:	*no attack*	
defense:	*(no defense)*	

assertion:	$O\ A_1 \wedge A_2$	
attack:	$P\ \wedge_i$	($i = 1$ or 2)
defense:	$O\ A_i$	

assertion:		$P\ A \to B$
question:		O ?
choice:	$P\ \lvert A \to B \rvert$	$P\ C$ only if $O\ C \to (A \to B)$ before
attack:	$O\ A$	
defense:	$P\ B$	

Likewise for atoms a:

assertion:	$P\ a$
question:	O ?
choice:	$P\ C$ only if $O\ C \to a$ before

P/*O*-symmetry of argumentation forms is given up.

Implications as rules: Dialogues and strategies

Dialogues

(C') O can question a (complex or atomic) formula A if and only if
(i) A has not yet been asserted by O, or
(ii) A has already been attacked by P.

(D') P may assert an atomic formula without O having asserted it before.

(E) O can only react on the immediately preceding P-move.

(Strategies defined as before.)

Corresponds to sequent calculus with alternative schema

$$\frac{\Gamma \vdash A}{\Gamma, A \to B \vdash B}$$

Yields 'dialogical' interpretation of implications-as-rules concept.

Implications as rules: Example

0.	P	$(a{\rightarrow}b){\rightarrow}((b{\rightarrow}c){\rightarrow}(a{\rightarrow}c))$		
1.	O	?	question	
2.	P	$\lvert(a{\rightarrow}b){\rightarrow}((b{\rightarrow}c){\rightarrow}(a{\rightarrow}c))\rvert$	choice	
3.	O	$a{\rightarrow}b$	attack	assuming rule $b{\leftarrow}a$
4.	P	$(b{\rightarrow}c){\rightarrow}(a{\rightarrow}c)$	defense	

Implications as rules: Example

0.	P $(a \to b) \to ((b \to c) \to (a \to c))$		
1.	O ?	question	
2.	P $\lvert (a \to b) \to ((b \to c) \to (a \to c)) \rvert$	choice	
3.	O $a \to b$	attack	(assuming rule $b \leftarrow a$)
4.	P $(b \to c) \to (a \to c)$	defense	
5.	O ?	question	
6.	P $\lvert (b \to c) \to (a \to c) \rvert$	choice	
7.	O $b \to c$	attack	assuming rule $c \leftarrow b$
8.	P $a \to c$	defense	

Implications as rules: Example

0.	P	$(a \to b) \to ((b \to c) \to (a \to c))$		
1.	O	?	question	
2.	P	$\lvert (a \to b) \to ((b \to c) \to (a \to c)) \rvert$	choice	
3.	O	$a \to b$	attack	(assuming rule $b \leftarrow a$)
4.	P	$(b \to c) \to (a \to c)$	defense	
5.	O	?	question	
6.	P	$\lvert (b \to c) \to (a \to c) \rvert$	choice	
7.	O	$b \to c$	attack	(assuming rule $c \leftarrow b$)
8.	P	$a \to c$	defense	
9.	O	?	question	
10.	P	$\lvert a \to c \rvert$	choice	
11.	O	a	attack	
12.	P	c	defense	

Implications as rules: Example

0.	P	$(a \to b) \to ((b \to c) \to (a \to c))$		
1.	O	?	question	
2.	P	$\lvert (a \to b) \to ((b \to c) \to (a \to c)) \rvert$	choice	
3.	O	$a \to b$	attack	(assuming rule $b \leftarrow a$)
4.	P	$(b \to c) \to (a \to c)$	defense	
5.	O	?	question	
6.	P	$\lvert (b \to c) \to (a \to c) \rvert$	choice	
7.	O	$b \to c$	attack	(assuming rule $c \leftarrow b$)
8.	P	$a \to c$	defense	
9.	O	?	question	
10.	P	$\lvert a \to c \rvert$	choice	
11.	O	a	attack	
12.	P	c	defense	
13.	O	?	question	
14.	P	b	choice	using rule $c \leftarrow b$

Implications as rules: Example

0.	$P\ (a{\to}b){\to}((b{\to}c){\to}(a{\to}c))$		
1.	$O\ ?$	question	
2.	$P\ \lvert(a{\to}b){\to}((b{\to}c){\to}(a{\to}c))\rvert$	choice	
3.	$O\ a{\to}b$	attack	(assuming rule $b{\leftarrow}a$)
4.	$P\ (b{\to}c){\to}(a{\to}c)$	defense	
5.	$O\ ?$	question	
6.	$P\ \lvert(b{\to}c){\to}(a{\to}c)\rvert$	choice	
7.	$O\ b{\to}c$	attack	(assuming rule $c{\leftarrow}b$)
8.	$P\ a{\to}c$	defense	
9.	$O\ ?$	question	
10.	$P\ \lvert a{\to}c\rvert$	choice	
11.	$O\ a$	attack	
12.	$P\ c$	defense	
13.	$O\ ?$	question	
14.	$P\ b$	choice	(using rule $c{\leftarrow}b$)
15.	$O\ ?$	question	
16.	$P\ a$	choice	using rule $b{\leftarrow}a$

Implications as rules: Example

0.	P	$(a{\to}b){\to}((b{\to}c){\to}(a{\to}c))$		
1.	O	?	question	
2.	P	$\lvert(a{\to}b){\to}((b{\to}c){\to}(a{\to}c))\rvert$	choice	
3.	O	$a{\to}b$	attack	(assuming rule $b{\leftarrow}a$)
4.	P	$(b{\to}c){\to}(a{\to}c)$	defense	
5.	O	?	question	
6.	P	$\lvert(b{\to}c){\to}(a{\to}c)\rvert$	choice	
7.	O	$b{\to}c$	attack	(assuming rule $c{\leftarrow}b$)
8.	P	$a{\to}c$	defense	
9.	O	?	question	
10.	P	$\lvert a{\to}c\rvert$	choice	
11.	O	a	attack	
12.	P	c	defense	
13.	O	?	question	
14.	P	b	choice	(using rule $c{\leftarrow}b$)
15.	O	?	question	
16.	P	a	choice	(using rule $b{\leftarrow}a$)

O cannot question $P\,a$ due to (C'): a asserted by O before and not attacked by P.
Dialogue is won by P and is a strategy for $(a{\to}b){\to}((b{\to}c){\to}(a{\to}c))$.

Implications as rules and Cut

No 'Cut-elimination', but subformula property.

Argumentation form for Cut:

assertion:	$O\ A$	(or O ?, . . .)
attack:	$P\ \ B$	
defense:	$O\ B$	

Implications as rules and Cut

No 'Cut-elimination', but subformula property.

Argumentation form for Cut:

	assertion:	$O\ A$	(or O ?, . . .)
	attack:	$P\ \ B$	
	defense:	$O\ B$	

0.	$P\ \ a{\rightarrow}((a{\rightarrow}(b{\wedge}c)){\rightarrow}b)$		
1.	O ?	[0, question]	
2.	$P\ \ \lvert a{\rightarrow}((a{\rightarrow}(b{\wedge}c)){\rightarrow}b)\rvert$	[1, choice]	
3.	$O\ a$	[2, attack]	
4.	$P\ \ (a{\rightarrow}(b{\wedge}c)){\rightarrow}b$	[3, defense]	
5.	O ?	[4, question]	
6.	$P\ \ \lvert(a{\rightarrow}(b{\wedge}c)){\rightarrow}b\rvert$	[5, choice]	
7.	$O\ a{\rightarrow}(b{\wedge}c)$	[6, attack]	(assuming rule $(b{\wedge}c){\leftarrow}a$)

Implications as rules and Cut

No ‘Cut-elimination’, but subformula property.

Argumentation form for Cut:	assertion:	$O\ A$	(or O ?, . . .)
	attack:	$P\ \ B$	
	defense:	$O\ B$	

0.	$P\ \ a{\rightarrow}((a{\rightarrow}(b{\wedge}c)){\rightarrow}b)$		
1.	O ?	[0, question]	
2.	$P\ \ \lvert a{\rightarrow}((a{\rightarrow}(b{\wedge}c)){\rightarrow}b)\rvert$	[1, choice]	
3.	$O\ a$	[2, attack]	
4.	$P\ \ (a{\rightarrow}(b{\wedge}c)){\rightarrow}b$	[3, defense]	
5.	O ?	[4, question]	
6.	$P\ \ \lvert(a{\rightarrow}(b{\wedge}c)){\rightarrow}b\rvert$	[5, choice]	
7.	$O\ a{\rightarrow}(b{\wedge}c)$	[6, attack]	(assuming rule $(b{\wedge}c){\leftarrow}a$)
8.	$P\ \ b{\wedge}c$	[Cut]	

Implications as rules and Cut

No ‘Cut-elimination’, but subformula property.

Argumentation form for Cut:

assertion:	O A	(or O ?, . . .)
attack:	P B	
defense:	O B	

0.	$P\ \ a\to((a\to(b\wedge c))\to b)$		
1.	O ?	[0, question]	
2.	$P\ \ \lvert a\to((a\to(b\wedge c))\to b)\rvert$	[1, choice]	
3.	$O\ a$	[2, attack]	
4.	$P\ \ (a\to(b\wedge c))\to b$	[3, defense]	
5.	O ?	[4, question]	
6.	$P\ \ \lvert(a\to(b\wedge c))\to b\rvert$	[5, choice]	
7.	$O\ a\to(b\wedge c)$	[6, attack]	(assuming rule $(b\wedge c)\leftarrow a$)
8.	$P\ \ b\wedge c$	[Cut]	
9.	O ?	[8, question]	
10.	$P\ \ a$	[9, choice]	using rule $(b\wedge c)\leftarrow a$
11.			
12.			

Implications as rules and Cut

No ‘Cut-elimination’, but subformula property.

Argumentation form for Cut:

	assertion:	$O\ A$	(or O ?, . . .)
	attack:	$P\ \ B$	
	defense:	$O\ B$	

0.			$P\ \ a{\to}((a{\to}(b{\wedge}c)){\to}b)$		
1.			O ?	[0, question]	
2.			$P\ \ \lvert a{\to}((a{\to}(b{\wedge}c)){\to}b)\rvert$	[1, choice]	
3.			$O\ a$	[2, attack]	
4.			$P\ \ (a{\to}(b{\wedge}c)){\to}b$	[3, defense]	
5.			O ?	[4, question]	
6.			$P\ \ \lvert(a{\to}(b{\wedge}c)){\to}b\rvert$	[5, choice]	
7.			$O\ a{\to}(b{\wedge}c)$	[6, attack]	(assuming rule $(b{\wedge}c){\leftarrow}a$)
8.			$P\ \ b{\wedge}c$	[Cut]	
9.	$O\ \ b{\wedge}c$	[Cut]	O ?	[8, question]	
10.	$P\ \ \wedge_1$	[9, attack]	$P\ \ a$	[9, choice]	(using rule $(b{\wedge}c){\leftarrow}a$)
11.	$O\ b$	[10, defense]			
12.	$P\ \ b$	[7, defense]			

Conclusions

(i) Dialogical extension for clausal definitions (databases/logic programs).

(ii) Implications treated as rules.

(iii) In dialogical treatment *P*/*O*-symmetry of argumentation forms is given up.