

The categorical and the hypothetical

Some remarks

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Proof and Dialogues
Tübingen, 25–27th February, 2011

Outline

- 1 Categorical, Hypothetical and Functional concepts
- 2 A case for the primacy of the functional layer
- 3 Remarks on the dialogical approach

The priority of the Categorical over to Hypothetical

Logical and Material Consequence

$$A \vDash B =_{\text{def}} \forall \mathfrak{B}, A \vDash_{\mathfrak{B}} B$$

The transmission view of consequence

$$A \vDash_{\mathfrak{B}} B =_{\text{def}} \vDash_{\mathfrak{B}} A \Rightarrow \vDash_{\mathfrak{B}} B$$

The truth-theoretic case

$$A \vDash_{\mathcal{M}} B =_{\text{def}} \vDash_{\mathcal{M}} A \Rightarrow \vDash_{\mathcal{M}} B$$

The proof-theoretic case

$$A \vDash_{\mathfrak{G}} B =_{\text{def}} \exists f \forall c \vDash_{\mathfrak{G}} c \left\{ \begin{array}{c} \nabla \\ A \end{array} \right\} \Rightarrow \vDash_{\mathfrak{G}} f(c) \left\{ \begin{array}{c} \nabla \\ B \end{array} \right\}$$

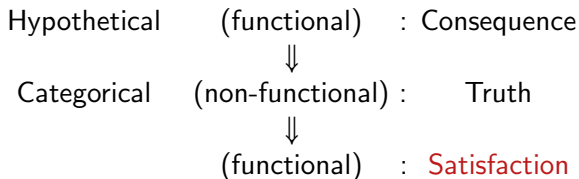
Are there other semantic concepts whose nature is functional?
In which relationship do they stand to categorical concepts?

The truth-theoretic case (II)

Truth and satisfaction

$$\models_{\mathcal{M}} A =_{\text{def}} \forall \sigma, \sigma \models_{\mathcal{M}} A$$

where A may in general be an open formula
(i.e. a sentential *function*)



The semantics is grounded on a basic functional concept

The proof-theoretic case (II)

Implication

$$\models_{\mathcal{G}} \begin{array}{c} \nabla \\ A \rightarrow B \end{array} \stackrel{=_{def}}{\models_{\mathcal{G}}} \begin{array}{c} A \\ \nabla \\ B \end{array}$$

where $\begin{array}{c} A \\ \nabla \\ B \end{array}$ may in general be an open argument

Curry-Howard

If $x : A$ and $t : B$ then $\lambda x.t : A \rightarrow B$

Open assumptions \Leftrightarrow Free variables

Open derivations \Leftrightarrow Open terms

Is this functional concept at the core of the proof-theoretic semantics?

The proof-theoretic case (III)

Open arguments

$$\vDash_{\mathfrak{G}} \begin{array}{c} A \\ \nabla \\ B \end{array} =_{\text{def}} \forall \mathfrak{G}' \supset \mathfrak{G}, \vDash_{\mathfrak{G}'} \begin{array}{c} \nabla \\ A \end{array} \Rightarrow \vDash_{\mathfrak{G}'} \begin{array}{c} \nabla \\ A \\ \nabla \\ B \end{array}$$

$$\vDash_{\mathfrak{G}} \begin{array}{c} A \\ \nabla \\ B \end{array} \Leftrightarrow A \vDash_{\mathfrak{G}} B$$

Hypothetical (functional) : Consequence/Open Argument
 \Downarrow
 Categorical (non-functional) : Closed argument

Refutation-theoretic approach

Reversing the direction of transmission

$$B \vDash_{\mathfrak{G}} A \stackrel{=_{\text{def}}}{=} \exists f \forall c \vDash_{\mathfrak{G}} c \left\{ \begin{array}{c} A \\ \Delta \end{array} \right\} \Rightarrow \vDash_{\mathfrak{G}} f(c) \left\{ \begin{array}{c} B \\ \Delta \end{array} \right\}$$

The dual of implication

$$\vDash_{\mathfrak{G}} \begin{array}{c} A \multimap B \\ \Delta \end{array} \stackrel{=_{\text{def}}}{=} \vDash_{\mathfrak{G}} \begin{array}{c} B \\ \Delta \\ A \end{array} \stackrel{=_{\text{def}}}{=} \forall \mathfrak{G}' \supset \mathfrak{G}, \vDash_{\mathfrak{G}'} \begin{array}{c} A \\ \Delta \end{array} \Rightarrow \vDash_{\mathfrak{G}'} \begin{array}{c} B \\ \Delta \\ A \\ \Delta \end{array}$$

Functional first

Proofs and refutations as limit cases

$$\begin{array}{c}
 A \\
 \diamond \\
 B
 \end{array}
 \begin{array}{l}
 \rightsquigarrow \\
 \\
 \rightsquigarrow
 \end{array}
 \begin{array}{c}
 A \\
 \triangle \\
 \\
 \nabla \\
 B
 \end{array}
 \rightsquigarrow
 B \models A =_{\text{def}}
 \begin{array}{c}
 A \\
 \triangle \\
 \\
 \nabla \\
 A
 \end{array}
 \Rightarrow
 \begin{array}{c}
 B \\
 \triangle \\
 \\
 \nabla \\
 B
 \end{array}$$

Which model for such a primitive notion?

Sequent calculus (or clausal definitions):
the basis of a bi-directional model of reasoning

Lorenzen-style dialogues

Play and strategy

- The play level is characterized by the interaction of the two players
- The strategy level is grounded on the one of play

Winning strategy for P (proofs)
—and possibly for O (refutations)—
defined through the more primitive
notion of play

Can the notion of play be displayed as functional, or hypothetical?

References



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College Publications, 2009;



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a critique of some fundamental assumptions of standard semantics

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Natural deduction for dual-intuitionistic logic

Studia Logica, accepted.