The categorical and the hypothetical

Some remarks

Luca Tranchini

Wilhelm-Schickard-Institut
Universität Tübingen

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Outline

1. Categorical, Hypothetical and Functional concepts
2. A case for the primacy of the functional layer
3. Remarks on the dialogical approach

The priority of the Categorical over to Hypothetical

Logical and Material Consequence

$$A \vDash B =_{def} \forall \mathfrak{B},\ A \vDash_{\mathfrak{B}} B$$

The transmission view of consequence

$$A \vDash_{\mathfrak{B}} B =_{def} \vDash_{\mathfrak{B}} A \Rightarrow \vDash_{\mathfrak{B}} B$$

The truth-theoretic case

$$A \vDash_{\mathscr{M}} B \ =_{def} \vDash_{\mathscr{M}} A \ \Rightarrow \vDash_{\mathscr{M}} B$$

The proof-theoretic case

$$A \vDash_{\mathfrak{S}} B \ =_{def} \ \exists f \forall c \ \vDash_{\mathfrak{S}} c \left\{ \begin{array}{c} \bigtriangledown \\ A \end{array} \right. \Rightarrow \vDash_{\mathfrak{S}} f(c) \left\{ \begin{array}{c} \bigtriangledown \\ B \end{array} \right.$$

Are there other semantic concepts whose nature is functional?
In which relationship do thay stand to categorical concepts?

The truth-theoretic case (II)

Truth and satisfaction

$$\vDash_{\mathscr{M}} A \ =_{def} \ \forall \sigma, \ \sigma \vDash_{\mathscr{M}} A$$

where A may in general be an open formula
(i.e. a sentential *function*)

$$\begin{array}{lcl} \text{Hypothetical} & \text{(functional)} & : \text{Consequence} \\ & \Downarrow & \\ \text{Categorical} & \text{(non-functional)} & : \text{Truth} \\ & \Downarrow & \\ & \text{(functional)} & : \text{Satisfaction} \end{array}$$

The semantics is grounded on a basic functional concept

The proof-theoretic case (II)

Implication

$$\vDash_{\mathfrak{G}} \begin{array}{c} \bigtriangledown \\ A \to B \end{array} \quad =_{def} \vDash_{\mathfrak{G}} \begin{array}{c} A \\ \bigtriangledown \\ B \end{array}$$

where $\begin{array}{c} A \\ \bigtriangledown \\ B \end{array}$ may in general be an open argument

Curry-Howard

If $x : A$ and $t : B$ then $\lambda x.t : A \to B$

Open assumptions $\Leftrightarrow$ Free variables   Open derivations $\Leftrightarrow$ Open terms

Is this functional concept at the core of the proof-theoretic semantics?

The proof-theoretic case (III)

Open arguments

$$\vDash_{\mathfrak{S}} \begin{array}{c} A \\ \bigtriangledown \\ B \end{array} =_{def} \forall \mathfrak{S}' \supset \mathfrak{S}, \; \vDash_{\mathfrak{S}'} \begin{array}{c} \bigtriangledown \\ A \end{array} \Rightarrow \vDash_{\mathfrak{S}'} \begin{array}{c} \bigtriangledown \\ A \\ \bigtriangledown \\ B \end{array}$$

$$\vDash_{\mathfrak{S}} \begin{array}{c} A \\ \bigtriangledown \\ B \end{array} \Leftrightarrow A \vDash_{\mathfrak{S}} B$$

Hypothetical	(functional)	:	Consequence/Open Argument
		$\Downarrow$	
Categorical	(non-functional)	:	Closed argument

Refutation-theoretic approach

Reversing the direction of transmission

$$B \vDash_{\mathfrak{S}} A \;=_{def}\; \exists f \forall c \;\; \vDash_{\mathfrak{S}} c \left\{ \begin{array}{c} A \\ \triangle \end{array} \right. \;\Rightarrow\; \vDash_{\mathfrak{S}} f(c) \left\{ \begin{array}{c} B \\ \triangle \end{array} \right.$$

The dual of implication

$$\vDash_{\mathfrak{S}} \begin{array}{c} A \prec B \\ \triangle \end{array} \;=_{def}\; \vDash_{\mathfrak{S}} \begin{array}{c} B \\ \triangle \\ A \end{array} \;=_{def}\; \forall \mathfrak{S}' \supset \mathfrak{S},\; \vDash_{\mathfrak{S}'} \begin{array}{c} A \\ \triangle \end{array} \;\Rightarrow\; \vDash_{\mathfrak{S}'} \begin{array}{c} B \\ \triangle \\ A \\ \triangle \end{array}$$

Functional first

Proofs and refutations as limit cases

$$\begin{array}{c}A\\ \Diamond\\ B\end{array}$$

$$\nearrow \begin{array}{c}\mathsf{A}\\ \triangle\end{array} \rightsquigarrow B \vDash A =_{def} \begin{array}{c}\mathsf{A}\\ \triangle\end{array} \Rightarrow \begin{array}{c}\mathsf{B}\\ \triangle\end{array}$$

$$\searrow \begin{array}{c}\triangledown\\ \mathsf{B}\end{array} \rightsquigarrow A \vDash B =_{def} \begin{array}{c}\triangledown\\ \mathsf{A}\end{array} \Rightarrow \begin{array}{c}\triangledown\\ \mathsf{B}\end{array}$$

Which model for such a primitive notion?

Sequent calculus (or clausal definitions):
the basis of a bi-directional model of reasoning

Lorenzen-style dialogues

Play and strategy

- The play level is characterized by the interaction of the two players
- The strategy level is grounded on the one of play

Winning strategy for P (proofs)
—and possibly for O (refutations)—
defined through the more primitive
notion of play

Can the notion of play be displayed as functional, or hypothetical?

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