Remarks on Game-Based Theories of Meaning

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Outline

1. Theories of meaning
2. Hintikka's GTS / Dummett's anti-realism
3. Dialogical logic and GTS
4. Proof-conditional semantics
5. Conclusion

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Truth-conditional theory of meaning

- The truth-conditions of S: the different alternative circumstances under which S is true.
- Meaning of S determines a function

$$f_S : \mathcal{C} \to \{0, 1\},$$

with $f_S(c) = 1$ iff S is true at c

- Realism: sentences possess an objective truth-value, independently of our means of knowing the truth-value.

Grasping the meaning (truth-conditional)

- *To understand S is to know what is the case if S is true.*

(LW: *TLP* 4.024)

When presented with a circumstance c, I must be able to say whether S is true at c or not.

Example

Suppose c_0 comprises an infinity of objects $a_1, a_2, \ldots$ each of which is Q. If presented with c_0, grasping the meaning of $\forall xQx$ allows me to say that this sentence is true at c_0.

It's totally irrelevant that I might have insurmountable difficulties in *being presented* with c_0 (i.e., finding out that the 'actual world' is structured as c_0.)

Anti-realist critique of the truth-conditional view

- Anti-realism (a.k.a. justificationism, verificationism).
- Basic notion *recognizing as true* rather than *being true*.
- Meaningful to ascribe truth to S only in circumstances c in which we have a means of recognizing its truth.

Example

Let c_0 be as above. According to the anti-realist, we cannot meaningfully ascribe truth to $\forall xQx$: given our human limitations, we lack means of recognizing its truth.

- *Understanding S consists in an ability to recognize, when suitably placed, whether S is true or false.*

(Dummett: *TR*, 59)

Anti-realist critique (cont.)

- Anti-realist: Specification of truth-conditions does not suffice to yield meaning.
- We may agree that learning the meaning of *S* does not happen via such a specification. But this does not preclude that the meaning, once mastered, can be so described.
- Anti-realist: How could we possibly learn to apply 'true' to sentences *S* in circumstances *c* in which we have no way of recognizing that *S* is true?
  - This critique suggests that we learn to apply the word 'true' sentence by sentence, circumstance by circumstance.
  - But arguably truth is not a matter of an unanalyzed comparison of *S* itself with *c* — rather the concept emerges via the semantic roles of the syntactic components of *S*.

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Hintikka's game-theoretic semantics (a.k.a. GTS)

- The notions of truth and meaning are explicated by means of certain sorts of (model-relative) games.
- The resulting semantics is truth-conditional **and** (in an abstract sense) verificationist.
- The truth-conditions are defined in terms of the very activities of verification and falsification.
- 'Verification' not in the sense of Dummett's 'justification.'

Semantic games (general)

- Model-relative two-player games: 'semantic games.'
  - Two players (say 1 and 2),
  - Two roles (*verifier* or $\mathbb{V}$, *falsifier* or $\mathbb{F}$);
    role distributions $\rho : \{\mathbb{V}, \mathbb{F}\} \to \{1, 2\}$.
- The rules are meant to create links between language and the 'reality' (a model).
- The relevant actions *witnessing* and *instantiating*.
  - **Level of plays**: seeking and finding
  - **Level of strategies**: verification and falsification
- A is true (*resp.* false) in $\mathcal{M}$: there is a winning strategy for player 2 (*resp.* player 1) in the semantic game $G(A, \mathcal{M})$.

Semantic game $G(A, \mathcal{M})$:

- **Initial position:** $\langle A, \mathcal{M}, \rho_0 \rangle$, with $\rho_0(\mathbb{V}) = 2$ and $\rho_0(\mathbb{F}) = 1$.
- **Game rules:** Suppose $\langle B, \mathcal{M}, \rho \rangle$ is a position.
  - If $B = \exists x D$, player $\rho(\mathbb{V})$ selects an individual and names it (say n). The play continues with the position $\langle D[x/n], \mathcal{M}, \rho \rangle$.
  - If $B = (C_1 \vee C_2)$: player $\rho(\mathbb{V})$ chooses a disjunct C_i.
  - If B is $\forall x D$ or $(C_1 \wedge C_2)$: as above but $\rho(\mathbb{F})$ makes the move.
  - If $B = \neg C$, the players switch roles: the play continues with the position $\langle C, \mathcal{M}, \rho^* \rangle$, where ρ^* is the transposition of ρ.
  - If B is atomic, the play ends and $\mathcal{M}$ determines the payoffs: $\rho(\mathbb{V})$ wins if B true in $\mathcal{M}$, otherwise $\rho(\mathbb{F})$ wins.

Truth, meaning, understanding

- It is stressed that we get two things at the price of one: Once the play level is fixed, so is the strategy level.
- Meaning does not presuppose the notion of truth: the meanings of logical operators and the notion of truth (applied to complex sentences) are constituted together.
- Understanding sentences requires mastering certain activities: **knowing how to play** certain games.
- Language users **do not** themselves **play** these games.

GTS verificationist — in which sense?

- Verifications$_1$:
  - means of gaining knowledge / means of recognizing truth.
  - prerequisite for truth ascriptions for an anti-realist.
  - epistemic aspect.
- Verifications$_2$:
  - winning strategies of the initial *verifier* is semantic games.
  - objective; encode 'combinatorial' facts about the model.
  - have nothing to do with epistemic efforts.
- The existence of a verification$_2$ does not require the existence of a verification$_1$.
- Verifications$_1$ implement verifications$_2$ or are their epistemically accessible realizations.

GTS compared with anti-realism

Example (infinite domain, $S := \forall x(Bx \rightarrow Cx)$)

A-R: The assertibility conditions of S cannot be satisfied: we cannot possess means of recognizing the requisite infinity of facts. No verification$_1$ exists.

GTS: The semantically relevant actions serve to associate the quantifier $\forall x$ with a single object in an infinite domain.

The truth of S is not a matter of a one-time ascription whose justification is subject to our limitations. Verification$_2$ exists.

Example (finite domain, $S := \exists xBx$)

A-R: The truth of S is recognized by inspecting the elements until one is found out to be B. Verification$_1$ yields knowledge.

GTS: Verification$_2$ of $\exists xBx$ consists of selecting a certain object a_i. Knowledge of the truth of $\exists xBx$ is another matter.

GTS: summary

- Middle ground between
  - variants of truth-conditional semantics which take the notion of truth as an unanalyzed basic concept, and
  - the verificationist views laying stress on the epistemic capacities of the language users.
- There are no separate language games for 'truth.'
- We do not learn to apply the notion of truth case by case, depending on the sort of sentence and the sort of circumstances at hand.

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Semantic games and material dialogues

- How do Hintikka's semantic games relate to what can be formulated in the dialogical framework?
- Setting aside the philosophical ideas related to GTS *resp.* to DL, semantic games can be construed as dialogues.
- Consider the syntax of FO with the operators $\vee, \wedge, \neg, \exists, \forall$.
- In DL, we consider a structural rule stipulating that the players choose at the beginning of a play *repetition ranks*.
  - If a player has chosen rank k, she may attack any given utterance at most k times and defend a given utterance against a fixed attack at most k times.

Strict material dialogues

- A model is assumed to be given.
- The particle rules are as in formal dialogues, except that in the quantifier rules, it is understood that for any object in the domain a constant symbol may be introduced.
- Structural rules modified as follows:
  - Repetition ranks of both players equal 1 (strictness).
  - The winning rule: whoever utters a false atomic sentence, or cannot move, has lost, while the adversary has won.
  - Material dialogues have no formal rule.
  - Makes no difference whether the 'intuitionistic rule' or the 'classical rule' is adopted.
- A is true (GTS) in $\mathcal{M}$ iff there is a w.s. for **P** in the strict material dialogue $\mathcal{D}(A)$ relative to $\mathcal{M}$.

Comparison with GTS

- Strict material dialogues are the dialogical counterpart of Hintikka's semantic games.
- By strictness, a player must always react to the immediately preceding move by the adversary.
- Moreover, the immediately preceding move uniquely determines to which *sentence* the player must react.
- The length of a play of $\mathcal{D}(A)$ is at most $2 \cdot N$, where N is the maximum number of nested logical operators in A.
- As soon as a player utters an atom, the play ends.
- Note: the dialogical distinction between **P** and **O** corresponds to two distinctions in GTS: the two possible role distributions *and* the two players 1 and 2.

Generalizations

- From the dialogical viewpoint, semantic games can be generalized in various ways — retaining the particle rules.
  - Giving up strictness: allowing arbitrary ordinal numbers as repetition ranks.
  - Giving up model-relativity: towards a characterization of validity (logical truth).
  - Enriching the language (notably adding $\rightarrow$ to the syntax).
- Theoretical benefit of DL: offering a "uniform analysis" of material truth and validity.
- Note: Technically DL captures the perfectly objective, realist notion of "truth in a model."

Dialogues and anti-realism

- Are there any grounds for associating DL with anti-realism?
- Do only those sentences come out materially/logically true for which we possess means of recognizing them as such?
  - **No**: the existence of a winning strategy for **P** in a dialogue has nothing to do with our epistemic restrictions.
  - In material dialogues winning strategies spell out objective truth-conditions.
  - Surely, a language user taking the place of **P** may not master a winning strategy while one exists.
  - But this is not an argument for anti-realism — trivially some truths are not known to a given person in a given context.

Dialogues and anti-realism (cont.)

- Anti-realism might creep into DL notably via criteria for winning a given (terminal) play.
- Yet, this suggests non-ascribability of truth only due to atomic sentences.
- Unlike with Dummett, in DL a sentence like $\forall xBx$ cannot fail to be true (in the sense of DL itself) if all 'instances' of Bx are individually recognized as being true.
- DL represents at most a quite mild version of anti-realism.
- And the realist can utilize the DL framework: after all, it's one thing to win a play and another to know to have won it!

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Proof-conditional semantics

- Basic notions: proof, constructive procedure.
  - Basic notions in dialogues: types of moves.
- Meanings of logical operators explicated in terms of the notion of proof.
- Lays down how proofs of complex sentences are related to proofs of certain syntactically less complex sentences.
- Already the basic semantic notion is of strategic character.
  - being provable cf. the existence of a w.s. for **P**
  - a proof object / proof cf. a w.s. for **P**
  - no counterpart to the play level.

Proof-conditional semantics (cont.)

- The corresponding semantic maneuver in DL would be to suggest that meanings of logical operators are defined in terms of winning strategies.
- In DL, however, it is maintained that meanings of these expressions is defined at the play level.
- The play level allows a level of analysis not available in proof-conditonal semantics.
- Learning the meaning of the logical operators:
  - **Dummett:** By being *trained* to assert complex statements on certain kinds of situations. We cannot extract from this training more than was put into it.
  - **GTS/DL:** By learning the correlated game rules.

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- The dialogical approach locates meaning in the play level.
- Semantic games: technically dialogues of a special kind.
- Hintikka's philosophical motivation for GTS is free from anti-realism. Yet the resulting theory of meaning is (not only truth-conditional but also) in a sense verificationist.
- Only a mild anti-realism seems to be motivated by DL.
- Proof-conditional semantics operates with 'strategic notions' (proof, constructive procedure).
    - Unlike GTS/DL, it appears not to recognize a more fundamental level of meaning constitution.