

Section 5: Logic & Scientific Method

POPPER'S INFERENCE DEFINITIONS OF LOGICAL CONSTANTS

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This paper attempts to shed some new light on Popper's little-known articles of 1947-49 on the foundations of (deductive) logic.<sup>1</sup> These articles suffer from the fact that they were written without awareness of the state of the art in mathematical logic and, in particular, of Gentzen type inference systems. They nevertheless contain ideas which are particularly interesting from a more modern perspective, and which deserve to be better known.

Popper's framework is based on an inference relation which essentially has the structural features of Gentzen's sequent arrow (identity, weakening and cut). Logical operations are defined metalinguistically by the inferential role they play, independently of whether they are syntactically represented by means of a connective. For example, an (arbitrarily formed) sentence  $A$  is called a *disjunction of  $B$  and  $C$* , if for any  $D$ :  $D$  can be inferred from  $A$  if and only if  $D$  can be inferred from  $B$  as well as from  $C$ .

These definitions are not to be understood as a new sort of semantics. A semantics would start with a formal language, define a central semantical notion for its sentences such as truth, and *justify* an inference relation on the basis of such a definition. Rather, given an already established inference relation, an inferential definition *singles out* certain operations by calling them conjunctions, disjunctions, negations etc. of sentences.

We shall argue that this idea is highly original, in spite of the flaws in Popper's presentation. It is closely related to modern attempts to specify logical constants or logical systems in terms of consequence or implication relations<sup>2 3</sup>, and in particular to Koslow's structuralist theory of logic<sup>4</sup>. We shall also compare Popper's characterization of the underlying inference relation with ideas developed by Hertz and Gentzen in the 1920s and 1930s.<sup>5</sup>

Although inferential definitions in Popper's sense can be a powerful *descriptive* tool, in particular when different logical systems are investigated, they seem to us not suited to provide a *foundation* for logic (if there is such a thing at all). We shall discuss in detail the interrelationship between inferential definitions, semantical considerations and questions concerning the logicity of operations.

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<sup>1</sup> K.R. Popper, *New foundations for logic*, *Mind* 56 (1947), 193-235, and five other papers. See the bibliography in: P. Schroeder-Heister, *Popper's theory of deductive inference and the concept of a logical constant*, *History and Philosophy of Logic*, 5 (1984), 79-110.

<sup>2</sup> P. Schroeder-Heister, *Structural frameworks, substructural logics, and the role of elimination inferences*. In: G. Huet & G. Plotkin (eds.), *Logical Frameworks*, Cambridge 1991, 385-403.

<sup>3</sup> D. Gabbay (ed.), *What is a Logical System?* Oxford 1994.

<sup>4</sup> A. Koslow, *A Structuralist Theory of Logic*, Cambridge 1992.

<sup>5</sup> P. Schroeder-Heister, *Resolution and the origins of structural reasoning: Early proof-theoretic ideas of Hertz and Gentzen*, *Bulletin of Symbolic Logic* (to appear).