

- PETER SCHROEDER-HEISTER, *An alternative implication-left schema for the sequent calculus.*

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As an alternative to Gentzen's schema  $(\rightarrow L)$  for the introduction of implication on the left side of the sequent sign in the intuitionistic sequent calculus **LJ** we propose the schema  $(\rightarrow L)^\circ$ :

$$(\rightarrow L) \frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \rightarrow B \vdash C} \quad (\rightarrow L)^\circ \frac{\Gamma \vdash A}{\Gamma, A \rightarrow B \vdash B}$$

In the absence of cut,  $(\rightarrow L)^\circ$  is weaker than  $(\rightarrow L)$ . In the system based on  $(\rightarrow L)^\circ$ , cut is admissible except for cuts whose left premiss is the conclusion of  $(\rightarrow L)^\circ$ , i.e., cuts of the following restricted form:

$$(\rightarrow L)^\circ \frac{\begin{array}{c} \vdots \\ \Gamma \vdash A \end{array} \quad \begin{array}{c} \vdots \\ A, \Delta \vdash C \end{array}}{(\text{cut}) \frac{\quad}{\Gamma, \Delta \vdash C}}$$

Using cut in this restricted form,  $(\rightarrow L)$  and  $(\rightarrow L)^\circ$  can be shown to be equivalent. Unlike full cut, applications of restricted cut do not compromise the subformula property and are harmless in this sense. Philosophically,  $(\rightarrow L)^\circ$  is motivated by the interpretation of implications as rules [1, 2] and can be viewed as a direct translation of *modus ponens* into the sequent calculus.

[1] P. SCHROEDER-HEISTER, *Generalized elimination inferences, higher-level rules, and the implications-as-rules interpretation of the sequent calculus*, **Advances in Natural Deduction** (E. H. Haeusler, L. C. Pereira and V. de Paiva, editors), 2010.

[2] ——— *Implications-as-rules vs. implications-as-links: An alternative implication-left schema for the sequent calculus*, submitted for publication.